# Homework Day 7 Solutions - ECON 186

**Problem 1.** Chiang and Wainwright 9.2 #1

1)

a) $f'(x) = -4x + 8 = 0$ iff $x = 2$.
   $f'(1.99) = 0.04 > 0$, $f'(2.01) = -0.04 < 0$
So the stationary value $f(2) = 15$ is a relative maximum.

b) $f'(x) = 10x + 1 = 0$ iff $x = -\frac{1}{10}$
   $f'\left(-\frac{1}{9}\right) = -0.11 < 0$, $f'\left(-\frac{1}{11}\right) = 0.09 > 0$
So the stationary value $f\left(-\frac{1}{10}\right) = -\frac{1}{20}$ is a relative minimum.

c) $f'(x) = 6x = 0$ iff $x = 0$
   $f'(-.01) = -0.06 < 0$, $f(.01) = 0.06 > 0$
So the stationary value $f(0) = 3$ is a relative minimum.

   d) $f'(x) = 6x - 6 = 0$ iff $x = 1$
   $f'(.99) = -0.06 < 0$, $f'(1.01) = 0.06 > 0$
   So the stationary value $f(1) = -1$ is a relative minimum.

**Problem 2.** For problem 1, parts (a) and (b), determine whether each function is concave or convex (strictly?)

a)
$$f''(x) = -4$$

So the function is strictly concave.

b)
$$f''(x) = 10$$

So the function is strictly convex.

**Problem 3.** Chiang and Wainwright 9.3 #1(b, d), 6

1)

b)

$$f'(x) = 28x^3 - 3$$
$$f''(x) = 84x^2$$
$$f'''(x) = 168x$$

d)

$$f'(x) = 2(1-x)^{-2}$$
$$f''(x) = 4(1-x)^{-3}$$
$$f'''(x) = 12(1-x)^{-4}$$

6)

a) Linear utility function

b) We know that the expected utility from playing is
$$EU = 0.5 \times U(\$10) + 0.5 \times U(\$20)$$

For risk neutral people, their utility is proportional to the amount of dollars that they have, so $0.5 \times U(\$10) = U(\$5)$, etc. So,
$$EU = 0.5 \times U(\$10) + 0.5 \times U(\$20) = U(\$5) + U(\$10) = U(\$15)$$

So $U(\$15)$ is the expected utility for a risk neutral person. Note that the expected utility for a risk neutral person is just the utility they get from having the expected value of the game.

**Problem 4.** Chiang and Wainwright 9.4 #1, 3(b, c, d, e), 6

1)

a) $f'(x) = -4x + 8 = 0$; $f''(x) = -4$. The critical value is $x^* = 2$; the stationary value $f(2) = 33$ is a maximum.

b) $f'(x) = 3x^2 + 12x = 0$; $f''(x) = 6x + 12$. The critical values are 0 and $-4$. $f(0) = 9$ is a minimum, because $f''(0) = 12 > 0$, but $f''(-4) = 41$ is a maximum, because $f''(-4) = -12 < 0$.

c) $f'(x) = x^2 - 6x + 5 = 0$; $f''(x) = 2x - 6$. The critical values are 1 and 5. $f(1) = \frac{16}{3}$ is a maximum because $f''(1) = -4$, but $f(5) = -\frac{16}{3}$ is a minimum because $f''(5) = 4$.

d) $f'(x) = \frac{2}{(1-2x)^2} \neq 0$ for any value of $x$; there exists no relative extremum.

3)

2

b) First, we get the average revenue function
$$P = 100 - Q$$

Then, we have
$$R = PQ = (100 - Q)Q = 100Q - Q^2$$

c)
$$\pi = R - C = -\frac{1}{3}Q^3 + 6Q^2 - 11Q - 50$$

d)
$$\frac{d\pi}{dQ} = -Q^2 + 12Q - 11 = 0$$
This yields two critical values, $Q^* = 1$ and $Q^* = 11$.

e) Recall that the second order condition for profit maximization is
$$R^{''}(Q) \leq C^{''}(Q)$$

In this example,
$$R^{''}(Q) = -2$$
$$C^{''}(Q) = 2Q - 14$$

So, the condition for a maximum is only satisfied for the critical value $Q^* = 11$. Then, the maximum profit is
$$\pi^* = -\frac{1}{3}(11)^3 + 6(11)^2 - 11(11) - 50 = -\frac{1331}{3} + 726 - 121 - 50 = \frac{1665 - 1331}{3} = \frac{334}{3}$$

6)

a) Production function: $Q = f(L)$
Revenue function: $R = P_0Q = P_0f(L)$
Cost function: $C = W_0L + F$
Profit function: $\pi = R - C = P_0f(L) - W_0L - F$

b)
$$\frac{d\pi}{dL} = P_0f^{'}(L) - W_0 = 0 \rightarrow P_0f^{'}(L) = W_0$$

The value of the marginal product must be equal to the wage rate. Economically, this makes sense because the marginal product (how much the workers are producing on the margin) is multiplied by what they can sell the product for, and this should be how much they get paid.

c)
$$\frac{d^2\pi}{dL^2} = P_0f^{''}(L)$$

Since $P_0$ is not 0, it must be that $f''(L) < 0$ for profit to be maximized. This means that there is a diminishing marginal product of labor.

**Problem** 5. Chiang and Wainwright 11.2 #1, 4, 5

1) The partial derivatives are: $f_x = 2x + y$, $f_y = x + 4y$, $f_{xx} = 2$, $f_{yy} = 4$, and $f_{xy} = 1$. The first-order condition requires that $2x + y = 0$ and $x + 4y = 0$. Thus we have $x^* = y^* = 0$ implying $z^* = 3$.

Second-order conditions: $f_{xx} = 2 > 0$, $f_{yy} = 4 > 0$, and $f_{xx}f_{yy} = 8 > f_{xy}^2 = 1$, so $z^* = 3$ is a minimum.

4) $f_x = 2(e^{2x} - 1)$, $f_y = 4y$, $f_{xx} = 4e^{2x}$, $f_{yy} = 4$, and $f_{xy} = 0$. The first order condition requires that $2ax = 0$ and $2by = 0$. Thus $x^* = y^* = 0$ so that $z^* = 4$.

Second-order conditions: Evaluated at the critical value, $f_{xx} = 4$, $f_{yy} = 4$, $f_{xy} = 0$, so $f_{xx}f_{yy} > f_{xy}^2$, so $z^* = 4$ is a minimum.

5)

a) Any pair $(x, y)$ other than $(2, 3)$ yields a positive $z$ value.

b) The first order conditions are

$$f_x = 4(x - 2)^3 = 0$$
$$f_y = 4(y - 3)^3 = 0$$

Evaluated at the critical value $(2, 3)$, we get

$$f_x = 4(2 - 2)^3 = 0$$

$$f_y = 4(3 - 3)^3 = 0$$

c) No. At $x^* = 2$ and $y^* = 3$, we have $f_{xx} = f_{yy} = f_{xy} = f_{yx} = 0$.

d) By (11.6), $d^2z = 0$. Thus (11.9) is satisfied. So, $z^* = 0$ satisfies the necessary, but not sufficient condition for a minimum.

**Problem 6.** Chiang and Wainwright 11.3 #4(a, d), 5(a, d)

4)

a)
$$q = \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

d)

$$q = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

5)

a)

$$3 > 0$$

$$\begin{vmatrix} 3 & -2 \\ -2 & 7 \end{vmatrix} = 21 + 4 = 25 > 0$$

So $q$ is positive definite.

d)

$$-2 < 0$$

$$\begin{vmatrix} -2 & 3 \\ 3 & -5 \end{vmatrix} = 10 - 9 = 1 > 0$$

So $q$ is negative definite.