Intergenerational Persistence in Education and Income:

An Evolutionary View[*]

Jijian Fan [†]   Hugo Lhuillier [‡]

November 5, 2018

Abstract

Despite the fact that education may increase one's lifetime income, intergenerational persistence in educational attainment is widely observed. We develop a model to explain the natural formation of intergenerational persistence in education and analyze the evolutionary dynamics. While the gains from education are the same for different subgroups of the population, constant education costs generate higher disutility for lower-income families. Under competitive labor supply, where increasing the size of the educated population reduces the payoff, lower-income families will give up education, which leads to intergenerational persistence. In a simple setup we find neutral stability with perfect immobility of education levels, but the share of educated population may differ. The implications are: (a) intergenerational persistence exists naturally; (b) partial subsidy of one-price education leads to social efficiency, reduces inequality and promotes welfare, yet immobility still exists; and (c) social persistence could be solved by price discrimination.

Keywords: Education; Household income; Social mobility; Evolutionary game.

JEL classification: I20, J62, C73.

1 Introduction

In any meritocratic society, the income and social position of everyone is said to reflect its own intelligence and effort throughout his or her life. Meritocratic societies emphasize particularly education as one of the best ways to climb the social ladder. Indeed, investment in education is a powerful tool to increase one's wage. Figure 1 shows the trend of difference in median earnings between workers with a high school degree and workers with a college degree, both showing the high return that yields investment in education. Within a perfectly equitable educational system, wage is supposed only to reflect difference in education attainment, and therefore difference in education investment, with the same chance to succeed at the beginning of one's education regardless of the social background of each student.

Yet, this meritocratic ideal seems not to fit the empirical data in most developed countries. In reality, we observe that education attainment has large persistence. According to Choy (2001), college enrollment rates in the United States vary largely with parents' educational attainment. In 1999, 82 percent of students whose parents held a bachelors degree or higher enrolled in college immediately after finishing high school. However, the rate falls to 54 percent for those whose parents had completed high school but not college and even lower for those whose parents had less than a high school diploma (36 percent). The college enrollment also varies with household income. Figure 2 shows that the likelihood of going to college coming from a favorable social background is greater than the chance of pursing superior education when your parents are poor. Furthermore, even though both proportions increased since the 1970s, the gap between high and low income households remained around 30% over time.

This paper, based on model from Restuccia and Urrutia (2004) on educational choice and using evolutionary game theory, studies the microeconomic foundation behind education attainment and its consequences on educational persistence.

The parents' decision to increase their childrens human capital through education can be considered as an investment. In the United States, attending superior education represents an important part of one's budget. Hence, parents and students are facing a tradeoff between working, or postponing their entry in the labor market by going to college, a decision involving costs and benefits. Because the return to education of one's agent depends on the strategy of the rest of the population, this decision can be modeled through evolutionary game theory.

First, a one-population model is set up, with two alternative strategies for the household making the entire population: invest or not in their childrens education. Then, in order to take into account the effect of households income in the decision of parents to invest in their children education, a two-population game is presented, one population being low-income household, the other one high-income household, once more with the two previously described alternative strategies.

The rest of paper is organized as follows. Section 2 describes the previous literatures on household education and education choice; Section 3 shows the theoretical framework and analyse the equilibrium and the dynamics of this game. Section 4 simulates this model and presents the main results, as well as some public policies implication; Section 5 discusses the limits of this model, and the possible way to expand it; and Section 6 concludes.

2 Literature Review

One of the most significant contributions to household income and family decisions was Becker (1962), in which he set up the first microeconomic model studying how people take decision about human capital investment with uncertainty on return to education. According to the author, this may for instance depend on factors such as *(a) Endowments/ Children Ability* and *(b) The longevity of life*, where both of these are known a priori. Becker also develops in this paper the idea that borrowing money to invest in education may sometimes be hard as return to education is more uncertain than return to physical capital. Consequently, internal financing would therefore be more common, leading to wealthier families tending to invest more than poorer ones in education. Becker and Chiswick (1966) brings the idea that the return to education depends on the number of people having the same degree. This theme had been widely developed in the 1970s, with economists expressing their concerns about over-education. Following Freeman (1975), Duncan and Hoffman (1982) formalized the idea and showed that 40% of the workforce at that time had more education than their jobs require in the US, triggering the return to postsecondary education to be half the size of the return to an additional year of required education. Still regarding the return to education, Altonji and Dunn (1996) explores the effects of personal and school characteristics on estimates of the return to education. One of his mainly findings was that controlling for the specific high school the student attended has only a modest effect on the rate of return to education.

More recent literature adds some complexity to the early model. Becker and Barro (1988) and Barro and Becker (1989) construct dynamic models to analyze the correlation between family income, fertility and human capital investment. Manski (1989) retakes and develop the idea of uncertainty regarding the return to education by formalizing the fact that a student, when considering whether or not to enroll in superior education, does not if he will be able to graduate. Restuccia and Urrutia (2004) build a recursive model to illustrate the trade-offs between consumption and education under dynamic stochastic general equilibrium framework and finds large intergenerational persistence.

Because low-income family are more likely to be risk-averse than highly educated household, this uncertainty can be one of the explanation why education attainment is skewed depending on students social background. Another reason, more obvious, is the cost of attending postsecondary education. A huge part of the economic literature on education studied the role of the family in the creation of income inequality. Knight (1997) is the first to identify family as the principal social institution in a market economy that fosters income inequality, an idea developed further by Berhman et al. (1995), and explained for instance by Restuccia and Urrutia (2004) who show that half of the intergenerational correlation in earnings is due to parental investment in education.

Because education seems to be one of the main way through which inequalities are perpetuated, several studies tried to quantify the role of family in students education attainment. Parsons (1975) quantifies empirically the role of family in education, concluding that both the quantity and productivity of educational investments are significantly determined by family wealth, as well as parental schooling. More recently, Björklund and Jäntti (2009) finds that family background (including community and neighborhood influences) is one of the main determinants for income during adulthood. Analyzing different countries and different periods, they quantified such factors to account for 40-50% of long-run income in the US. In the same vein, Belzil and Hansen (2003) finds that household background variables account for 68% of the explained variations in schooling attainments in the US, bulk of which is explained by parents education. This analysis, if especially true in the US, also holds in other countries, Björklund et al. (2006), Björklund et al. (2007), Ermisch and Francesconi (2001) find the same correlation respectively in Sweden and in England. Moreover, the two former papers were assessing the relative impact of pre-birth and post-birth factors in education attainment, both eventually concluding that the two factors were contributors to intergenerational transmissions.

Social reproduction is however not supposed to be the only possible outcome of education system. Björklund and Jäntti (2009) shows that reproduction in education attainment decreased over time in

Sweden over the last fifty years. Hertz et al. (2007) looks at 50-year trends in economic attainment reproduction in 42 nations, finding eventually large regional differences in educational persistence. Because there is no reason that newborns in Argentina are per se less clever than newborns in Sweden, those cross-countries difference prove that intergenerational persistence of educational attainment can be reduce with well-designed policies.

For the mechanism of educational persistence, most literatures such as Willis and Rosen (1978), Flug et al. (1998) and Li (2007) throw light upon the role of financial constraint. The effect of substitution between consumption and education investment is less modeled and empirically tested.

Another branch of education economics addresses the difference education policies available and their respective effects. Dynarski (1999), using the end of the Social Security Student Benefit Program as a quasi-experiment, estimates that offering $1,000 of financial aid increased the probability of attending college by four percentage points. Similarly, Dynarski (2000) uses Georgias HOPE Scholarship to evaluate the impact of the aid on middle and upper income youths college attendance. Her study estimated that each $1,000 in aid increased the college attendance rate by 3.7 to 4.2 percentage points; however, as this aid is a non need-based program, the author also found that the program enlarged the gap in college attendance between low- and high-income families as well as between blacks and whites households.

Based on his model in Manski (1989), Manski discusses whether financial aid should be based on enrollment in or completion of postsecondary schooling, and conclude that graduation-contingent aid is more appealing only under a set of restrictive condition, including that the reason why students drop out is because they want to. On the contrary, enrollment-based scholarship aid are more appealing if students do not know whether they will be able to graduate, which occurs to be often the case for students coming from low-educated families.

Tempering previous results, Hansen (1983) and Kane (1995), studying low and high-income youth attendance after the introduction of Pell Grant, find no significant increase in the low-income youth attendance rate. Meanwhile, Kane (1994), using within and cross-variation in public university tuition, find decreasing by $1,000 tuition prices augmented young black people college attendance by 3.7 percentage points. In the same study, he also looked at the effect of aid eligibility on enrollment and found that its estimated impact was only a third of that of an equivalent reduction in tuition. Eventually, Hoxby (1996) proposes that the most efficient education policy is not categorical financial

aid but the attribution of vouchers for public schools that are transferable between districts or schools, allowing to neutralize neighborhood effect.

Economists have not yet analyzed the evolutionary dynamics of education attainment along with its policy implications. In this paper, by applying evolutionary game theory method, we would like to fucus on the dynamic properties of educational choice.

3 Theoretical Framework

To begin with, in Section 3.1 is considered a simplified model with homogenous parents. This helps understanding the trade-offs and the externality of education. In Section 3.2 a dynamic model with heterogenous income is considered. The equilibrium and dynamics are discussed in Section 3.3 and 3.4.

3.1 Simplified Model

Consider the following game of education: homogenous parents choose the level of education for their siblings. They can decide high type education $(H, e = 1)$ or lower education $(L, e = 0)$. The constant cost of H is denoted by τ and the cost of L is 0. However, the benefits from education depends on the relative competitiveness. The job market becomes more competitive when there are more people with the same education level, and less competitive when there are more people with different type of education. Hence the income of kids depends on whether they are educated, and the share of educated people, i.e.,

$$I = I(e, N_H, N_L)$$

where e, N_H and N_L denote respectively individual's education level, number of people with high education and number of people with low education. Specifically, let the production function take the Cobb-Douglas form,

$$Y = \tilde{A}K^{1-\alpha-\beta}N_H^\alpha N_L^\beta$$

Assume that the capital level remains constant, and define a constant parameter $A = \tilde{A}K^{1-\alpha-\beta}$ representing the production from total factor productivity and capital contribution. Normalizing the population to 1, then the production function takes a simplified form of share of high educated people, i.e.,

$$Y_t = As_H^\alpha s_L^\beta = As_H^\alpha(1 - s_H)^\beta$$

where $\alpha > \beta > 0$ denote for the output elasticities for high type and low type of labor, respectively. Note that the constant return to scale of Cobb-Douglas function leads to the decreasing return to scale of aggregate labor, i.e., $\alpha + \beta < 1$.

Assume for complete competitive market, where the wage income to marginal return with each individual's labor supply normalized to 1. Then, the benefit from education is derived as

$$I(e|s_H) = \begin{cases} A\alpha s_H^{\alpha-1}(1-s_H)^\beta & \text{if } e = 1, \\ A\beta s_H^\alpha(1-s_H)^{\beta-1} & \text{if } e = 0. \end{cases}$$

Therefore, the payoff is given by

$$w_H = A\alpha s_H^{\alpha-1}(1-s_H)^\beta - \tau$$
$$w_L = A\beta s_H^\alpha(1-s_H)^{\beta-1}$$

The equilibrium s_H^* is implicitly given by

$$\Delta w(s_H) \equiv w_H - w_L = A\alpha s_H^{\alpha-1}(1-s_H)^\beta - \tau - A\beta s_H^\alpha(1-s_H)^{\beta-1} = 0$$

Note that $s_H^* \in (0,1)$, i.e., there exists a unique equilibrium, and the downward-crossing[1] of Δw guarantees that this equilibrium is stable, which implies a Hawk and Dove-type (HD) game.

3.2 Dynamic Model Setup

This part will set up a formal dynamic model of education choice, following Restuccia and Urrutia (2004) in which heterogonously endowed parent makes decisions and cares about their children's expected utility. Yet, we will derive a simplified dynamic model by dropping uncertainty and innate ability, and focus on the evolutionary properties. The mechanism differs in that Restuccia and Urrutia (2004) regards education as human capital accumulation (increase in total amount of effective labor), but we will regard labors with different level of education as input factors with partial substitutability, which means that high educated people does *not necessarily* have higher wage than low-educated population.

---

[1]Because $\frac{d\Delta w}{ds} < 0$, $\lim\limits_{s \to 0^+} \Delta w(s) = +\infty$, and $\lim\limits_{s \to 1^-} \Delta w(s) = -\infty$. See Appendix A.1.

Assume that each generation lives for 1 period. In their lifetime, they work to earn income and give birth to next generation. In next period, they retire and leave the labor market, and their children grow up and become the new generation of labor. For simplicity, assume for a haploid world where one parent give birth to one kid, so that each generation has identical population and the demographic structure remain unchanged.

Assume that individual cannot hold capital or asset[2]. The only way for leaving wealth to kids is by education so that their income might be promoted. Current generation chooses consumption and education level for their kids to maximize the following recursive utility

$$V(I_t) = \max_{e_{t+1} \in \{0,1\}, c_t} \{u(c_t) + \delta \mathrm{E} V(I_{t+1})\}$$

where the lifetime direct utility from consumption $u(\cdot)$ takes form of constant relative risk aversion (CRRA)[3], that is,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

A constant parameter $\delta \in [0, +\infty)$ measures the level of care for children. $\delta = 0$ represent no care for kids and $\delta = \infty$ represent extreme care for kids. Budget constraint is given by

$$c_t + e_{t+1} \cdot \tau \le I_t$$

where c_t denotes lifetime consumption, and I_t is the income that depends on parent's pre-determined education level. The cost and benefit of education are the same as in Section 3.1. Thus parent's income is given by

$$I(e_t | s_{H,t}) = \begin{cases} A\alpha s_{H,t}^{\alpha-1}(1 - s_{H,t})^\beta & \text{if } e_t = 1, \\ A\beta s_{H,t}^\alpha (1 - s_{H,t})^{\beta-1} & \text{if } e_t = 0. \end{cases}$$

Rolling forward and we get the children's future income

---

[2]However, capital will appear in production function. An explanation for this setup is that, it is hard for wage-earners to accumulate enough capital and become capital owner. For the source of capital, assume that there exists the third population (capital holder) that do not join the game, but only holding capital and make a living on capital returns. This is consistent with Piketty and Goldhammer (2014).

[3]More generally, utility function should satisfy $u'(\cdot) > 0$, $u''(\cdot) < 0$, $\lim_{x \to 0^+} u'(0) = +\infty$. For the utility function, see discussion on preference in Section 5.

$$I(e_{t+1}|s_{H,t+1}) = \begin{cases} A\alpha s_{H,t+1}^{\alpha-1}(1 - s_{H,t+1})^{\beta} & \text{if } e_{t+1} = 1, \\ \\ A\beta s_{H,t+1}^{\alpha}(1 - s_{H,t+1})^{\beta-1} & \text{if } e_{t+1} = 0. \end{cases}$$

Therefore, the whole problem is determined with some proper initial values, and thus the evolution of educated people share could be solved.

This conclusion can be generalized to the case with multiple education level. For more information, see Appendix A.2.

3.3   Equilibrium

Parents regard their recursive utility as the payoffs. Thus it makes a two population[4] (high/low level educated parents), two strategy (high/low level education for kids, for each population) game. Notice that the income (which directly defines different populations) is affected by s_H; consequently, the equilibrium condition is not only the strategy of the two populations being unchanged, but also the share of educated people s_H remaining constant.

Given the share of educated population for next-generation, the payoff for population with income I_t is given by

$$w_H(I_t) = u(I_t - \tau) + \delta \mathrm{E}\left[V(I_{t+1}(1))\right]$$
$$w_L(I_t) = u(I_t) + \delta \mathrm{E}\left[V(I_{t+1}(0))\right]$$

The equilibrium is defined by strategy sets $\Pi_L \times \Pi_H$ and social state s_H, such that

$$w_\pi(I^i(s_H)) \geq w_{\tilde{\pi}}(I^i(s_H)), \forall \pi \in \Pi_i, i \in \{H, L\}$$

Generally, each population could have pure strategies or mixed strategy, which makes the game quite complicated. However, the following proposition shows that only one case is possible for equilibrium.

Proposition 1   In equilibrium, there is perfect social persistence of education, i.e., children always have the same education level as their parents.

---

[4]Strictly speaking, this is different from a typical setting of two-population game. We will discuss it in Section 3.4.

Proof   First, in equilibrium, s_H remains fixed so the incomes for different education level is fixed. Moreover, before considering the education cost, the return for higher educated individual should be strictly higher than return for lower educated individual. If not, there will be no incentive for education, which reduces the number of educated people and consequently increases the return for education.

Next, denote $\Delta u(I_t) \equiv u(I_t) - u(I_t - \tau)$, $\Delta E \equiv \mathrm{E}\left[V(I_{t+1}(1))\right] - \mathrm{E}\left[V(I_{t+1}(0))\right]$ and $\Delta w(I) \equiv w_H(I) - w_L(I)$, then,

$$\Delta w(I_t) = \delta \Delta E - \Delta u(I_t)$$

which means that the relative fitness of high-type education equals to the gain from education ($\delta \Delta E$) minus the loss of education ($\Delta u(I_t)$), in terms of utility. The gain from education comes from expected income increase of children, which is independent to parent's income level. However, the loss of education is defined by the disutility from reducing consumption, which varies with the parent income I_t. By concavity, $\Delta u'(\cdot) < 0$, indicating that the constant cost of education always brings higher utility loss for poor families than for rich families. Figure 3 shows the case. The same cost of education (in horizontal axis) leads to different utility loss (in vertical axis), and with concave utility function, utility loss for high-income parent is always lower.

Thus we have two implications. On one hand, mixed strategy equilibrium cannot exist, as the fact that $\Delta w(I^H) = \Delta w(I^L) = 0$ never holds eliminates the equilibrium where both population have mixed strategy. On the other hand, high-income people always have higher incentive for education than low-income people.

Recall that we can neither have two populations cannot playing same strategy, nor one pure with one mixed, in that s_H cannot remain fixed, and therefore, we will reach the only equilibrium, i.e., $e_{t+1} = e_t$ for $e_t = 0, 1$, with probability 1. **Q.E.D.**

Nevertheless, the uniqueness of strategy does not indicate the uniqueness of equilibrium share of educated population. Any value for s_H and $s_L = 1 - s_H$ that satisfies

$$w_H(I(1)) \geq w_L(I(1))$$
$$w_L(I(0)) \geq w_H(I(0))$$

would formalize an equilibrium. Therefore, even though the equilibrium strategies are identical, the equilibrium level of s_H^* may vary due to different initial values.

3.4 Evolutionary Dynamics

Although the equilibrium (or potential equilibria) seems straightforward, the dynamics is much more complicated than a normal 2×2 evolutionary game, in that we need not only track the share of strategies but also keep an eye on share of educated population. Specifically, a continuous-time evolutionary dynamics is given by

$$\dot{p} = p \cdot [w_H(I(1|s_H)) - \bar{w}_H]$$

$$\dot{q} = q \cdot [w_L(I(1|s_H)) - \bar{w}_L]$$

$$\dot{s}_H = \kappa[q(1 - s_H) - (1 - p)s_H]$$

where p and q denote the fraction of high and low educated parents who choose high education for their children, respectively. The first two equations follow the typical replicator dynamics, which represent a continuous-time limit of learning process according to Börgers and Sarin (1997). The third equation keeps track of s_H as a state variable. It is more intuitive to think about a discrete case where

$$s_{H,t+1} - s_{H,t} = q_t(1 - s_{H,t}) - (1 - p_t)s_{H,t}$$

This is to say, among the $s_{H,t}$ of high-educated parents, a fraction of p_t will continue giving their kids high education, and thus $(1-p_t)s_{H,t}$ share of population would escape from the high education group. Similarly, $q_t(1-s_{H,t})$ represents number of high-educated parents who give kids high education, thus it stands for the population newly join the high education group. The difference between them is therefore change of high education population. In continuous time, as we have assumed that the number and structure of population remain unchanged, within a very short period of time, if a certain number of people pass away, then there must be identical number of newborns. Denote κ as instantaneous crude birth/death rate, i.e., the ratio of number of birth/death to number of population in a short period of time. Then, for this small subset of people, a fraction of $q(1 - s_H) - (1-p)s_H$ represents the change of number of high-educated people. Since only κ share of population is "updated", we derive $\kappa[q(1 - s_H) - (1 - p)s_H]$ expressing the changing of high-educated share on total population, which forms the third equation of dynamics.

$\bar{w}_H$ and $\bar{w}_L$ stand for the weighted average payoff for high type and low type parents, i.e.,

$$\bar{w}_H \equiv p \cdot w_H(I(1|s_H)) + (1-p) \cdot w_H(I(0|s_H))$$

$$\bar{w}_L \equiv q \cdot w_L(I(1|s_H)) + (1-q) \cdot w_L(I(0|s_H))$$

In continuous time, the share of educated population s_H cannot change sharply, and thus parents would expect the future share of educated people same as current share. For simplicity, denote $I^H = I(1|s_H)$ and $I^L = I(0|s_H)$, Therefore, the dynamics is rewritten as a function of relative payoffs

$$\dot{p} = p \cdot (1-p) \cdot \Delta w(I^H)$$

$$\dot{q} = q \cdot (1-q) \cdot \Delta w(I^L)$$

$$\dot{s}_H = \kappa[q(1-s_H) - (1-p)s_H]$$

For a detailed derivation of the dynamic system, please refer to Appendix A.3.

Figure 4 shows the state space for the game of educational choice. The interaction of strategies made by two populations is represented by the square parallel to the page, with strategy of educated parent on the vertical axis and uneducated parent on horizontal axis.

To find the steady states, let $\dot{p} = \dot{q} = \dot{s}_H = 0$. Then,

$$\dot{p} = 0 \quad \Rightarrow \quad p = 0, \ p = 1, \ \text{or} \ s_H = (I^H)^{-1}(\Delta w^{-1}(0))$$

$$\dot{q} = 0 \quad \Rightarrow \quad q = 0, \ q = 1, \ \text{or} \ s_H = (I^L)^{-1}(\Delta w^{-1}(0))$$

$$\dot{s}_H = 0 \quad \Rightarrow \quad (1-p)s_H = q(1-s_H)$$

These three equation will give three hyperplane that sectionizes state space. Figure 8 shows a scratch of isoclines and dynamics. The equation $\dot{p} = 0$ gives $p = 0$ and $p = 1$, which are two planes with fixed p, and also gives the third plane that determines a constant s_H. These three planes are shown in (a). Similarly, (b) illustrates $\dot{q} = 0$ [5]. Lastly, equation $\dot{s}_H = 0$ solves for part of hyperbolic paraboloid in a unit cube, which is shown in (c).

---

[5]Notably, s_H from $\dot{q} = 0$ is less than s_H from $\dot{p} = 0$. The intuition is the same as Proof for Proposition 1.

Figure 8(d) shows the steady states. Not surprisingly, there is no interior steady state. Even if all the four edges ($p = 0$ or 1, $q = 0$ or 1) are steady states defined in evolutionary dynamics, they are not stable in fact. The difference comes from the fact that population with zero share extinct in evolutionary replicator dynamics, but strategy in our model may still exist. Instead, a *sink* is where our interests lies on. By analyzing the direction of movement around the steady states, it implies an existence of multiple equilibria with neutrally stability, which is exactly the same as in Section 3.3.

4 Simulation

To test the validity of model, we simulate the evolutionary dynamics with packages in R. The calibration is introduced in Section 4.1, followed by visualization of dynamics in Section 4.2. Next, we discuss the role of education cost and social welfare implication. Results are shown in Section 4.3. Last, comparative static on exogenous technology is shown in Section 4.4.

4.1 Calibration

There are seven parameters to calibrate: aggregate productivity A, productivity for high and low educated labor α and β, intergenerational discount rate δ, cost of education τ, absolute risk aversion σ that determines the shape of utility curve, and population updating rate κ.

Based on Weil (2012, chap. 6), assuming a production function with constant return to scale and three inputs: capital, raw labor and human capital, α and β are respectively quantified to 0.4 and 0.25. A has been set to 150, which is the combination of total factor productivity and capital. The cost of education τ is calibrated to be 20, in consistent with the fact that kids caring and education account for about one fourth to one third of lifetime income[6]. The intergenerational discount rate δ is set equal to 0.5, as converted from intergenerational discount factor[7]. Such setup implies that children expected utility matters, but less than parents' present consumption. σ has been set to 1 for simplicity[8]. Last, although κ is highly correlated with birth rate (under assumption of constant number and structure of population, $\kappa = \ln(\text{annual crude birth rate})$), we unfortunately cannot estimate for κ in reality due to the discrepancy of population structure; yet it does not matter much since any positive values of κ have identical dynamics, and thus we simply set κ equal to 1. Table 1

---

[6]Note that, this method overestimates the cost of education in that it assumes zero cost for raising a child without educating. Yet it is the best estimator at this stage, without empirical data.

[7]Annual discount rate is reported about 2.5%, and average year of having the first and the second child is around 25 and 30, an approximation is therefore taken by $0.975^{27.5} \approx 0.4984$

[8]Restuccia and Urrutia (2004) suggests that CES elasticity should be around 1.5. We leave this for further research.

summarizes the calibration

4.2 Dynamic Simulation

Figure 5, 6 and 7 shows the dynamics. It turns out that, as predicted by the theoretical solution, different initial states converges to same strategy, $(p, q) = (1, 0)$, but the steady state of educated population share s_H varies within a range.

4.3 Cost of Education

Table 2 shows the simulation result when coast of education varies. In conformity with our expectations, the steady state income of the highly educated is always higher than the ones of the low-educated workers, except when the cost of education is equal to zero, in which case there is no distinction between low and high-income household. Moreover, as expected, the share of high-educated increases as education becomes more affordable. Alongside, it takes longer and longer for the system to reach its steady state with no mobility as the cost of education gets lower and lower.

More than the pure cost of education, it is interesting to look at the cost of education in one household's budget. The simulation gives result close to empirical evidence: empirically, the cost of childrens education takes between a fourth and a third of a household lifetime income, therefore presuming a relatively quick convergence to the steady state with no social mobility. It is also clear that education is always relatively more expensive for low income household than for high-income families. Therefore, as education becomes less expensive, both the fraction of education cot in households' budget and the difference between high and low-income family shrink. This trend comes from the fact that the cost of education is cheaper, as well as because s_H increases, therefore reducing the income of highly educated as education becomes less scarce, and the gap between the two categories.

Note that at the limit, when education is free, there is no longer income disparity between educated and non-educated individuals, and the probability that high and low-educated parents invest in their children's education is close to their initial state.

Formally, for the welfare implication, consider about the social production maximization

$$\max_{s_H} A s_H^\alpha (1 - s_H)^\beta - \tau s_H$$

First order condition implies that

$$A\alpha s_H^{\alpha-1}(1-s_H)^{\beta} - A\beta s_H^{\alpha}(1-s_H)^{\beta-1} - \tau = 0$$

which is equivalent to

$$I^H - I^L = \tau$$

Therefore, the way to achieve social optimum is to *partially* subsidize the education by a lump-sum tax so that the income difference equals to internal cost of education. Otherwise, in total the population is undereducated. However, this means the perfect intergenerational immobility still exists.

4.4   Technology Improvement

The last question simulations help to answer is how does equilibrium change with technology? Two types of exogenous shock are considered.

Figure 9 shows the effect of changes in A on share of educated population s_H. Since A is defined as the composition of total factor productivity (TFP) and capital contribution in production, this could represent the case of either capital accumulation or TFP increases. Generally, the higher level of A leads to larger share of educated people. The intuition behind is, higher A decreases the loss from education cost in terms of utility, and thus people are more likely to give education to kids. Note that s_H is increasing at a decreasing rate and is therefore very unlikely to reach 1, even for very large values of A.

Figure 10 illustrates the effect of increase in relative productivity of high educated labor. We assume that total productivity from labor remains constant, but the productivity splits between high and low educated labor. Keeping $\alpha + \beta$ fixed, we find that an increase in α would increase the steady state share of educated population.

5   Discussion

We leave some problems about our model and implications for further discussion.

Multi-level   Under the 2-level-education setup with no uncertainty, the model leads to a natural persistence of education. A more generalized conclusion may be acquired from a multi-level-

education model with uncertainty, such as probability to success in education, incomplete signaling, family formation and intra-household specialization, etc. In general, there are many incomes states that follows a Markov process with transition matrix given. Education, then, becomes a modification of matrix. The main result will not be changed, only converting complete persistence to partial persistence.

Preference We assume that preference satisfies diminishing marginal utility, which should be consistent with risk aversion. However, it is likely that people with extremely low income have higher risk preference. If this expands to most of population, then the result may reverse, i.e., only low educated people choose education. Also, if we regard an S-shaped utility curve exists, the interior equilibrium may exist.

Cost of education In this paper we assume for constant educational cost, that is, no price discrimination for education. Yet in reality, most government subsidizes education for a certain group of people, which leads to the existence of interior equilibrium. The intuition goes as follows: if lower-income families enjoy a lower price, then the education cost in terms of utility loss is lower, and if it decreases to the same level as of high-income families, and the level is exactly same as benefits from children being educated, then it constructs a possibility for inner solution. See Appendix A.4.

Labor Production We assume that different types of education cannot fully substitute each other. Yet it sometimes is the case that a PhD may also do well in jobs that does not require education level (even though this forms a social waste). In this situation, high educated labor could fully substitute for low educated labor.

Aiming The aiming of education becomes another drawback. In this framework, we simply regard education as an investment to future productivity. What if people prefer education just because they want to learn (non-productive) knowledge? This kind of utilization is neglected. Moreover, technology accumulation is ignored so that equilibrium would be stable and there is no proper growth path.

Quality Since we didn't consider about the quality of education, it seems that complete subsidy solves all problems. However, in reality, it is observed that subsidy does harm to education quality in different channels. That make another problem on policy implication.

Borrowing Constraint The implication differs from previous literatures in aspect of borrowing constraint. Previous literatures such as Restuccia and Urrutia (2004) believes that borrowing constraints are the key source that explains why lower-income families invest less on education. However, this model shows that even without constraints, as long as the debt is paid by parents (instead of their children), the persistence still exists.

6 Conclusion

This paper built a microeconomic model to explain the logic behind parents' investment in their children's education. In simplified version, with household homogeneity in terms of income, we proved that a stable interior equilibrium exist, with a fraction of the population rationally remaining uneducated since, in competitive market, increasing population in high level education will reduce the return for education. When allowing for heterogeneity in household income, specifically when distinguishing between high and low-income household, we found that intergenerational persistence naturally exists, the reason being that, as return of education declines for a constant cost of education, the only households able to afford education for their children are high-income household.

In this model, with unique price of education, the aiming of achieving efficiency and eliminating intergenerational immobility cannot be solved simultaneously, as the former requires partial subsidy while the latter appeals for full subsidy. However, price discrimination may solve the problem. Also, it has indeed been shown that a lump-sum transfer decreasing only partially the cost of education will increase the social efficiency and average education level, but will not help to fight intergenerational persistence on education level as the steady state will still show a perfectly immobile economy.

By allowing disparity in cost of education, namely by subsidizing education only for low-income households, we think that it would be possible to reach a steady state with social mobility and positive cost of education. Empirical data, clearly reflecting intergenerational persistence on education level, let presume that this optimal cost of education has not been found yet.

References

Aaronson, D. (1998). Using sibling data to estimate the impact of neighborhoods on children's educational outcomes. *Journal of Human Resources*, 33(4):915–946.

Acemoglu, D. and Autor, D. (2009). Lectures in labor economics. *Unpublished manuscript, Department of Economics, Massachusetts Institute of Technology, Cambridge, MA. Retrieved from http://economics. mit. edu/files/4689.*

Akerlof, G. A. (1997). Social distance and social decisions. *Econometrica: Journal of the Econometric Society*, 65(5):1005–1027.

Altonji, J. G. and Dunn, T. A. (1996). The effects of family characteristics on the return to education. *The Review of Economics and Statistics*, 78(4):692–704.

Barro, R. J. and Becker, G. S. (1989). Fertility choice in a model of economic growth. *Econometrica: journal of the Econometric Society*, 57(2):481–501.

Becker, G. S. (1960). An economic analysis of fertility. In *Demographic and economic change in developed countries*, pages 209–240. Columbia University Press.

Becker, G. S. (1962). Investment in human capital: A theoretical analysis. *Journal of Political Economy*, 70(5):9–49.

Becker, G. S. and Barro, R. J. (1988). A reformulation of the economic theory of fertility. *The Quarterly Journal of Economics*, 103(1):1–25.

Becker, G. S. and Chiswick, B. R. (1966). Education and the distribution of earnings. *The American Economic Review*, 56(1):358–369.

Becker, G. S. and Lewis, H. G. (1974). Interaction between quantity and quality of children. In *Economics of the family: Marriage, children, and human capital*, pages 81–90. UMI.

Belzil, C. and Hansen, J. (2003). Structural estimates of the intergenerational education correlation. *Journal of Applied Econometrics*, 18(6):679–696.

Berhman, J. R., Pollak, R. A., and Taubman, P. (1995). *From Parent to Child*. University of Chicago Press.

Björklund, A. and Jäntti, M. (2009). Intergenerational income mobility and the role of family background. *Oxford Handbook of Economic Inequality, Oxford University Press, Oxford.*

Björklund, A., Jäntti, M., and Solon, G. (2007). Nature and nurture in the intergenerational transmission of socioeconomic status: Evidence from swedish children and their biological and rearing parents. *The B.E. Journal of Economic Analysis & Policy*, 7(2).

Björklund, A., Lindahl, M., and Plug, E. (2006). The origins of intergenerational associations: Lessons from swedish adoption data. *The Quarterly Journal of Economics*, 121(3):999–1028.

Börgers, T. and Sarin, R. (1997). Learning through reinforcement and replicator dynamics. *Journal of Economic Theory*, 77(1):1–14.

Choy, S. (2001). Students whose parents did not go to college: Postsecondary access, persistence, and attainment. Technical report, National Center for Education Statistics.

Duncan, G. J. and Hoffman, S. D. (1982). The incidence and wage effects of overeducation. *Economics of Education Review*, 1(1):75–86.

Durlauf, S. N. (2004). Neighborhood effects. *Handbook of Regional and Urban Economics*, 4:2173–2242.

Dynarski, S. (2000). Hope for whom? financial aid for the middle class and its impact on college attendance. Technical report, National bureau of economic research.

Dynarski, S. M. (1999). Does aid matter? measuring the effect of student aid on college attendance and completion.

Ermisch, J. and Francesconi, M. (2001). Family matters: Impacts of family background on educational attainments. *Economica*, 68(270):137–156.

Fan, C. S. and Zhang, J. (2013). Differential fertility and intergenerational mobility under private versus public education. *Journal of Population Economics*, 26(3):907–941.

Flug, K., Spilimbergo, A., and Wachtenheim, E. (1998). Investment in education: do economic volatility and credit constraints matter? *Journal of Development Economics*, 55(2):465–481.

Freeman, R. B. (1975). Overinvestment in college training? *Journal of Human Resources*, 10(3):287–311.

Galor, O. and Moav, O. (2002). Natural selection and the origin of economic growth. *The Quarterly Journal of Economics*, 197(4):1133–1191.

Galor, O. and Weil, D. N. (2000). Population, technology, and growth: From malthusian stagnation to the demographic transition and beyond. *The American Economic Review*, 90(4):806–828.

Griliches, Z. (1970). Notes on the role of education in production functions and growth accounting. In *Education, income, and human capital*, pages 71–128. NBER.

Hansen, W. L. (1983). Impact of student financial aid on access. *Proceedings of the Academy of Political Science*, 35(2):84–96.

Hertz, T., Jayasundera, T., Piraino, P., Selcuk, S., Smith, N., and Verashchagina, A. (2007). The inheritance of educational inequality: International comparisons and fifty-year trends. *The BE Journal of Economic Analysis & Policy*, 7(2):1–48.

Hoxby, C. M. (1996). Are efficiency and equity in school finance substitutes or complements? *The Journal of Economic Perspectives*, pages 51–72.

Kane, T. J. (1994). College entry by blacks since 1970: The role of college costs, family background, and the returns to education. *Journal of Political Economy*, 102(5):878–911.

Kane, T. J. (1995). Rising public college tuition and college entry: How well do public subsidies promote access to college? Technical report, National Bureau of Economic Research.

Kaushal, N., Magnuson, K., and Waldfogel, J. (2011). *How is family income related to investments in childrens learning?* Russell Sage Foundation.

Knight, F. H. (1997). *The ethics of competition*. Transaction Publishers.

Li, W. (2007). Family background, financial constraints and higher education attendance in china. *Economics of Education Review*, 26(6):724–734.

Lucas, S. R. (2001). Effectively maintained inequality: Education transitions, track mobility, and social background effects1. *American journal of sociology*, 106(6):1642–1690.

Manski, C. F. (1989). Schooling as experimentation: a reappraisal of the postsecondary dropout phenomenon. *Economics of education review*, 8(4):305–312.

Miller, P., Mulvey, C., and Martin, N. (1995). What do twins studies reveal about the economic returns to education? a comparison of australian and us findings. *The American Economic Review*, 85(3):586–599.

Parsons, D. O. (1975). Intergenerational wealth transfers and the educational decisions of male youth. *The Quarterly Journal of Economics*, 89(4):603–617.

Piketty, T. (2000). Theories of persistent inequality and intergenerational mobility. *Handbook of income distribution*, 1:429–476.

Piketty, T. and Goldhammer, A. (2014). *Capital in the twenty-first century*. Belknap Press.

Raaum, O., Salvanes, K. G., and Sørensen, E. Ø. (2006). The neighbourhood is not what it used to be. *The Economic Journal*, 116(508):200–222.

Restuccia, D. and Urrutia, C. (2004). Intergenerational persistence of earnings: The role of early and college education. *The American Economic Review*, 94(5):1354–1378.

Schultz, T. W. (1961). Investment in human capital. *The American economic review*, 51(1):1–17.

Solon, G., Page, M. E., and Duncan, G. J. (2000). Correlations between neighboring children in their subsequent educational attainment. *Review of Economics and Statistics*, 82(3):383–392.

Weil, D. (2012). *Economic Growth*. Addison-Wesley Longman Inc.

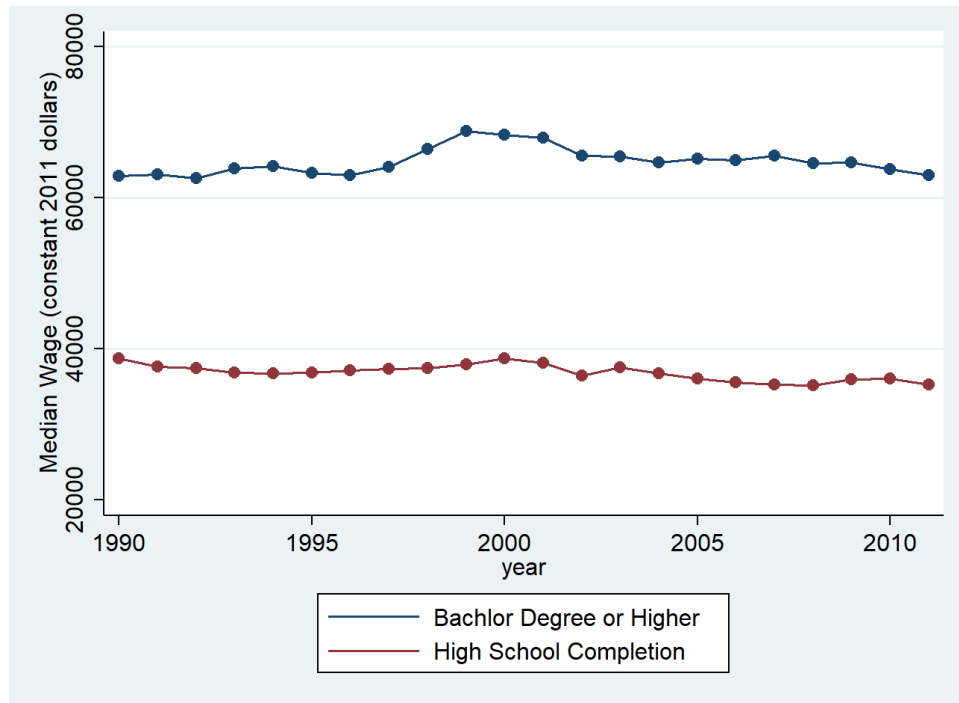Willis, R. J. and Rosen, S. (1978). Education and self-selection.

Figure 1: Median Wages in U.S., High School vs. College Degrees, 1990–2011

Notes: This figure shows the persistent difference between median annual wages of full-time workers over 25 years old. The wage is measured in 2011 constant US dollar. Source: NCES (2012) http://nces.ed.gov/programs/digest/d12/tables/dt12_438.asp
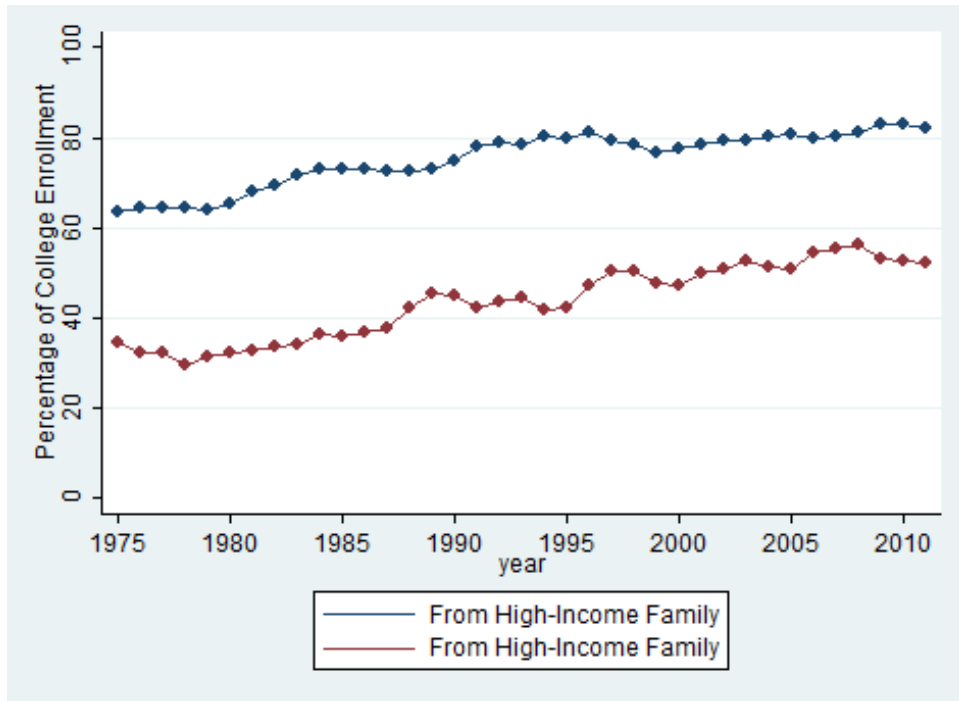
Figure 2: U.S. College Enrollment from Different Income Group, 1975–2011

Notes: This graph shows the college enrollment rates for offsprings from high- and low-income families. Source: NCES (2012) http://nces.ed.gov/programs/digest/d12/tables/dt12_236.asp
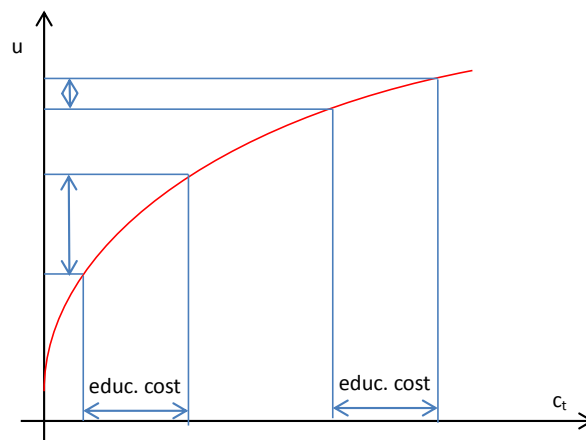

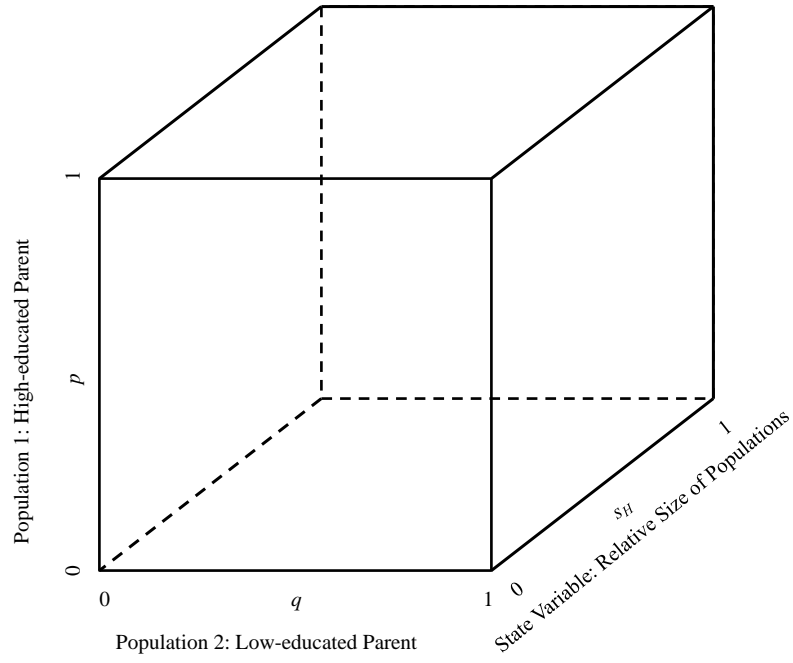Figure 3: Income Level Difference Leads to Different Loss of Education

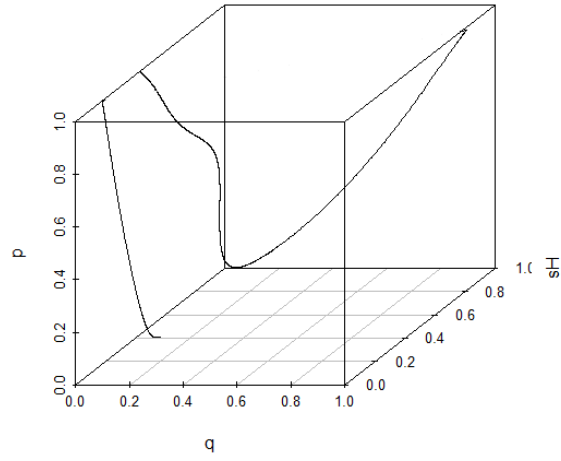Figure 4: The State Space for Educational Choice

Figure 5: Example of Dynamics in 3-D Space

Notes: The graph shows a dynamic path from initial $(p, q, s) = (0.05, 0.15, 0.3)$ and $(0.95, 0.95, 0.9)$.
Calibration: $\sigma = 1$, $A = 150$, $\alpha = 0.4$, $\beta = 0.25$, $\delta = 0.5$, $\tau = 20$, $\kappa = 1$.
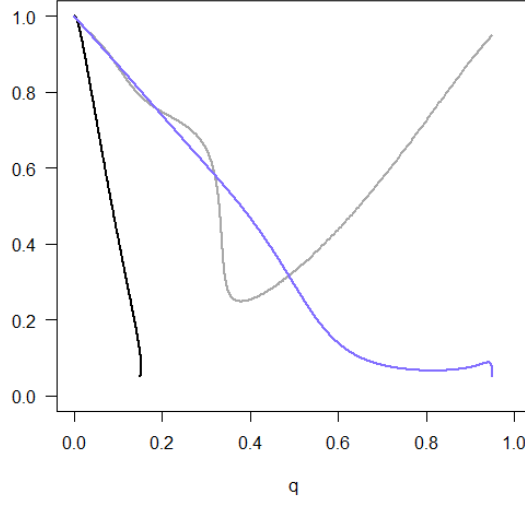
Figure 6: Evolutionary Change in Strategies

Notes: This graph shows a view from the front of Figure 5, i.e., projection on $p - q$ space. The graph shows dynamic paths from 5 different initial $(p, q, s) = (0.05, 0.15, 0.3)$, $(0.95, 0.95, 0.9)$ and $0.05, 0.95, 0.19)$. It indicates that different initial states may converge to same strategies. Calibration: $\sigma = 1$, $A = 150$, $\alpha = 0.4$, $\beta = 0.25$, $\delta = 0.5$, $\tau = 20$, $\kappa = 1$.
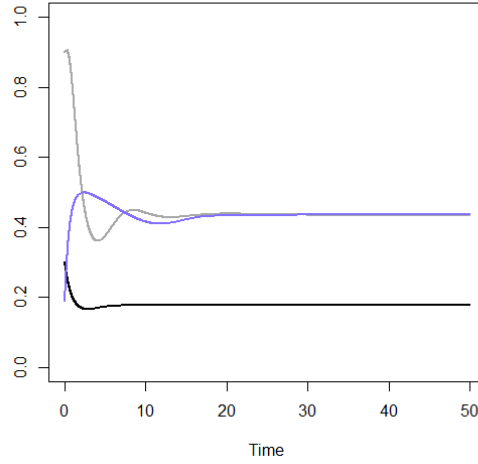

Figure 7: Evolutionary Change in Share of Educated Population

Notes: The graph shows the dynamics of education share s_H, i.e., projection of Figure 5 on s_H and keeping track of time. The graph shows dynamic paths from 5 different initial $(p, q, s) = (0.05, 0.15, 0.3)$, $(0.95, 0.95, 0.9)$ and $0.05, 0.95, 0.19)$. It indicates that even if equilibrium strategies are identical, different initial states may sink to equilibria with different values of s_H. Calibration: $\sigma = 1$, $A = 150$, $\alpha = 0.4$, $\beta = 0.25$, $\delta = 0.5$, $\tau = 20$, $\kappa = 1$.
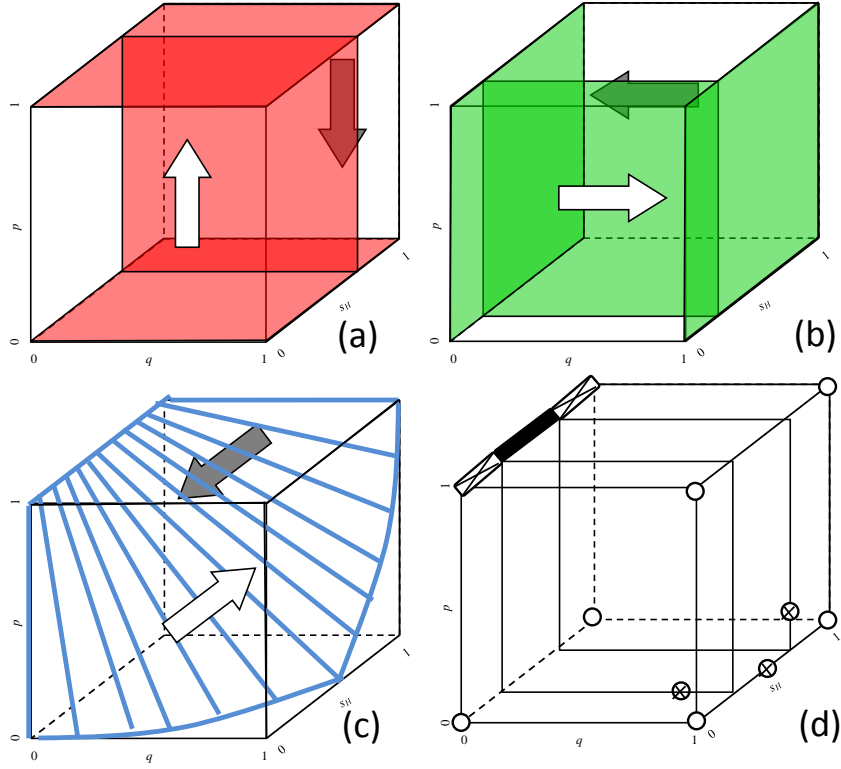
25

Figure 8: Isoclines, Direction of Movements and Steady States

Notes: Colored planes in (a)(b)(c) represent the isoclines for strategy p, q and state variable s_H. Arrows imply the direction of movement, where the light arrow locates above the page and dark arrow lies below the page. In (d) we show all steady states.
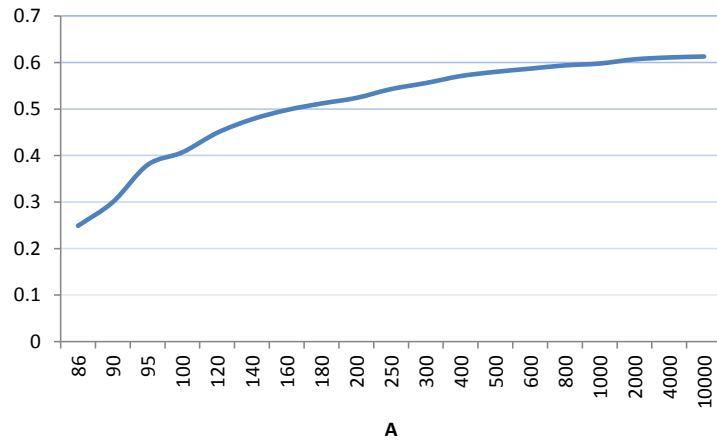

Figure 9: Effect of Capital Accumulation/TFP Improvement on Equilibrium Share of Education

Calibration: $\sigma = 1$, $\alpha = 0.4$, $\beta = 0.25$, $\delta = 0.5$, $\tau = 15$, $\kappa = 1$. Initial state: $(p, q, s_H) = (0.6, 0.4, 0.5)$.
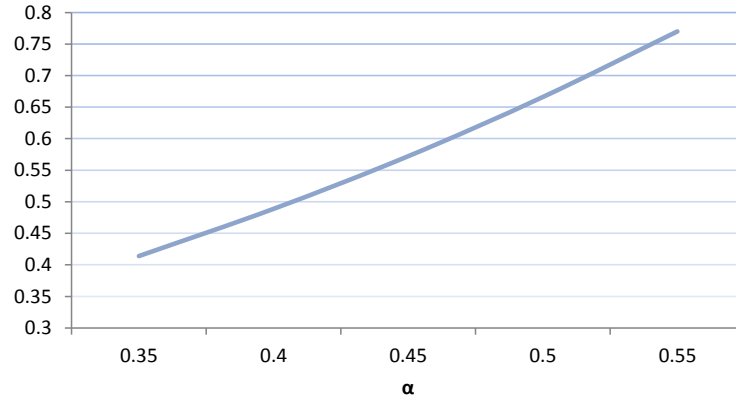
Figure 10: Effect of Education Productivity on Equilibrium Share of Education
Calibration: $\sigma = 1$, $A = 150$, $\alpha + \beta = 0.66$, $\delta = 0.5$, $\tau = 15$, $\kappa = 1$. Initial state: $(p, q, s_H) = (0.6, 0.4, 0.5)$.

Table 1: Simulation Parameter Calibration

Category	Variable	Value
Production	A	150
	α	0.4
	β	0.25
Preference	σ	1
	δ	0.5
Population updating	κ	1
Cost of Education	τ	20

Table 2: Equilibrium with Different Education Cost

τ^c	s_H^*	I_H^*	I_L^*	τ/I_H^*	τ/I_H^*	t^*
20	0.436	85.504	41.374	23.39%	48.34%	38
15	0.489	77.953	46.582	19.24%	32.20%	83
10	0.535	72.148	51.817	13.86%	19.30%	219
8	0.552	70.127	53.976	11.41%	14.82%	361
6	0.568	68.248	56.188	8.79%	10.68%	675
5	0.577	67.349	57.319	7.42%	8.72%	995
4	0.585	66.476	58.466	6.02%	6.84%	1,069
3	0.592	65.629	59.631	4.57%	5.03%	2,896
2	0.600	64.807	60.811	3.09%	3.29%	6,662
1	0.610	64.008	62.010	1.56%	1.61%	>16,665
0	0.610	63.230	62.230	0.00%	0.00%	N/A

Notes: Variables in each column stand for: cost of education faced by consumer (after subsidized), steady state share of educated population, steady state income for individual with high and low type of education, the ratio of education cost relative to their income, and convergent speed, respectively. The last column tracks the time period (ticks) it need to converge to $p_t > 0.9995$, and then the steady-state values are taken. When $\tau^c = 1$, the steady state values are taken at $t = 16,666$. There is no convergence for full subsidy, $\tau^c = 0$, instead, it has mixed strategy equilibrium. Calibration: $\sigma = 1$, $A = 150$, $\alpha = 0.4$, $\beta = 0.25$, $\delta = 0.5$, $\kappa = 1$. Initial state: $(p, q, s_H) = (0.6, 0.4, 0.5)$.

A   Appendices

A.1   Proof of HD-game properties

Since

$$\Delta w(s) = A\alpha s^{\alpha-1}(1-s)^\beta - A\beta s^\alpha (1-s)^{\beta-1} - \tau$$

$$= \left[As^{\alpha-1}(1-s)^{\beta-1}\right] \cdot \left[\alpha(1-s) - \beta s\right] - \tau$$

is continuous in $(0,1)$, and

$$\lim_{s\to 0^+} \Delta w(s) = \lim_{s\to 0^+} \left[As^{\alpha-1}(1-s)^{\beta-1}\right] \cdot \lim_{s\to 0^+} \left[\alpha(1-s) - \beta s\right] = +\infty \cdot \alpha = +\infty$$

$$\lim_{s\to 1^-} \Delta w(s) = \lim_{s\to 1^-} \left[As^{\alpha-1}(1-s)^{\beta-1}\right] \cdot \lim_{s\to 1^-} \left[\alpha(1-s) - \beta s\right] = +\infty \cdot (-\beta) = -\infty$$

Hence $\exists s^* \in (0,1)$ such that $\Delta w(s^*) = 0$ and $\frac{d}{ds}\Delta w(s)|_{s=s^*} < 0$, i.e., a downward-crossing equilibrium exists.

Further, notice that $s \in (0, 1)$, $\alpha \in (0, 1)$ and $\beta \in (0, 1)$, we can derive for the derivative as

$$
\begin{aligned}
\frac{d\Delta w(s)}{ds} &= \frac{d}{ds} \left\{ \left[As^{\alpha-1}(1-s)^{\beta-1} \right] \cdot [\alpha(1-s) - \beta s] - \tau \right\} \\
&= \left[As^{\alpha-1}(1-s)^{\beta-1} \right] \cdot \left(\frac{\alpha-1}{s} - \frac{\beta-1}{1-s} \right) \cdot [\alpha(1-s) - \beta s] + \left[As^{\alpha-1}(1-s)^{\beta-1} \right] \cdot (-\alpha - \beta) \\
&= \left[As^{\alpha-1}(1-s)^{\beta-1} \right] \cdot \left\{ -(\alpha + \beta) + \frac{(\alpha-1)(1-s) - (\beta-1)s}{s(1-s)} \cdot [\alpha(1-s) - \beta s] \right\} \\
&= \left[As^{\alpha-1}(1-s)^{\beta-1} \right] \cdot \left[-(\alpha + \beta) + \frac{\alpha(\alpha-1)(1-s)^2 + \beta(\beta-1)s^2}{s(1-s)} - \alpha(\beta-1) - \beta(\alpha-1) \right] \\
&= \left[As^{\alpha-1}(1-s)^{\beta-1} \right] \cdot \left[\frac{\alpha(\alpha-1)(1-s)^2 + \beta(\beta-1)s^2}{s(1-s)} - 2\alpha\beta \right] \\
&< 0
\end{aligned}
$$

Therefore, the equilibrium is unique. This proves a HD type game.

A.2 Possible Way for Generalization

A generalized version of the model is given by

$$
V(I_t) = \max_{e_t \in [0,1]} \left\{ u(c_t) + \delta \mathrm{E}\, V(I_{t+1}) \right\}
$$

subject to

$$
I_t = c_t + \tau(e_t)
$$

where $\tau(e)$ denotes the cost of education at level e. Assume that

$$
\tau(0) = 0, \quad \tau'(\cdot) > 0
$$

The Cobb-Douglas production function for labors in a continuum of education level is

$$
Y_t = \tilde{A} K^{1 - \int_0^1 \alpha(i)\,di} \int_0^1 N_{i,t}^{\alpha(i)}\,di
$$

$$
\alpha(i) > 0, \quad \alpha'(i) > 0, \quad \int_0^1 \alpha(i)\,di < 1
$$

Normalizing the population to 1 and the capital level to 1, then the production function takes a simplified form

$$
Y_t = A \int_0^1 s_{i,t}^{\alpha(i)}\,di
$$

and the income is given by

$$I(e|s_t) = A \cdot \frac{\alpha(e)}{s_{e,t}} \int_0^1 s_{i,t}^{\alpha(i)} di$$

Followed by a similar proof, it should be true that the equilibrium still takes the form that children always have the same education level as their parents. We leave this for a general further research.

A.3 Derivation for Dynamic System

First, assume that the default value function is calculated from the strategy that will fix on one type of education, i.e.,

$$V(I^H) = u(I^H - \tau) + \delta V(I^H)$$
$$V(I^L) = u(I^L) + \delta V(I^L)$$

which also gives the payoff functions for high type parents choosing high type education and for low type parents choosing low type education

$$w_H(I^H) = u(I^H - \tau) + \delta V(I^H) = \frac{1}{1 - \delta} u(I^H - \tau)$$
$$w_L(I^L) = u(I^L) + \delta V(I^L) = \frac{1}{1 - \delta} u(I^L)$$

Then, consider about a deviation: a deviation from current strategy means that the parents pick different education level, and suppose that their siblings would stay on that level of education. This gives

$$w_L(I^H) = u(I^H) + \delta V(I^L) = u(I^H) + \frac{\delta}{1 - \delta} u(I^L)$$
$$w_H(I^L) = u(I^L - \tau) + \delta V(I^H) = u(I^L - \tau) + \frac{\delta}{1 - \delta} u(I^H - \tau)$$

Taking difference, and we have the relative payoffs

$$\Delta w(I^H) \equiv w_H(I^H) - w_L(I^H) = \frac{1}{1 - \delta} u(I^H - \tau) - \frac{\delta}{1 - \delta} u(I^L) - u(I^H)$$
$$\Delta w(I^L) \equiv w_H(I^L) - w_L(I^L) = u(I^L - \tau) + \frac{\delta}{1 - \delta} u(I^H - \tau) - \frac{1}{1 - \delta} u(I^L)$$

The value of I^H and I^L comes from Section 3.1. To numerically solve for the differential equation system, we need calibration for: total factor productivity plus capital contribution A, factor

productivity for high educated labor α and low educated labor β, cost of education τ, measure of inter-generation care δ, and we also need to find a functional form for utility $u(\cdot)$.

A.4 Simulated Examples of Equilibria with Different Cost of Education

Table 3: Equilibrium with Different Asymmetric Education Cost

τ_H	τ_L	I_H^*	I_L^*	s_H^*	p^*	q^*
20	20	85.50	41.37	0.436	1	0
20	16	82.57	43.22	0.456	1	0
20	12	80.36	44.75	0.471	1	0
20	6	76.47	47.80	0.5	0	1
20	2	76.47	47.80	0.5	0	1
20	0	76.47	47.80	0.5	0	1

Notes: Variables in each column stand for: cost of education for high income household, cost of education for low income household, steady state income for individual with high and low type of education, steady state share of educated population, and the steady state of the probability that high-income household and low-income household invest in education of their children. Calibration is the same as before: $\sigma = 1$, $A = 150$, $\alpha = 0.4$, $\beta = 0.25$, $\delta = 0.5$, $\kappa = 1$. Initial state: $(p, q, s_H) = (0.6, 0.4, 0.5)$.

Table 3 show the result of simulations with assymetric cuts in the cost of education. The main conclusion of these simulations is that it exists a critical value of τ_L, $6 < \tau_L^* < 12$, under which the equilibria of the system change. Indeed, for $\tau_L > \tau_L^*$, the steady state of the system implies a perfectly immobile society, with only high-income household investing in education. However, when $\tau_L < \tau_L^*$, the system becomes "anti-immobile", with only low-income household investing in their children's education. As presented in Section 5, the reason is that, for $\tau_L < \tau_L^*$, τ_L being so low, and given the expected income driving by s_H, it is no longer profitable for high-income parents to invest in their children education. This simulation seems to be imply that there exist a value of τ, $\tau_L = \tau_L^*$, for which the system will have an interior steady state, or interior steady states.

Note that, as observed in the previous simulation with symmetric cuts of the cost of education, income of low-income household is still lower than the one of high-income household, mainly because the decrease in τ affects incomes only through the variation of s_H. Hence, as the cost of education decreases for poor families, education becomes more affordable and the share of high-educated increases, driving down the income of highly educated workers.

B Key Part of Coding

```
# This code is only for the system of odes
```

```r
library(deSolve)


Pars <- c(A <- 40, a <- .4, b <- .25, si <- .5, ta <- 3)

pars <- c(A <- 40, a <- .4, b <- .25, si <- .5, ta <- 3)

State <- c(p1 <- .3, p2 <- 0.7, q1 <- .1, q2 <- 0.9, sH <- .3)

times <- seq(0, 200, by = 0.01)


Educ_sim <- function(times, State, pars)

{

with(as.list(c(State, pars)), {

#Income function

Ih <- Pars[1] * Pars[2] * (State[5] ^ (Pars[2] - 1)) * ((1 - State[5])^Pars[3])

Il <- Pars[1] * Pars[3] * (State[5] ^ (Pars[2])) * ((1 - State[5])^(Pars[3] - 1))


#Payoff of high educated to give high education

w1 <- (1 / (1 - Pars[4])) * log(Ih - Pars[5])

#Payoff of high educated to give low education

w2 <- log(Ih) + (Pars[4] / (1 - Pars[4])) * log(Il)

#Payoff of low educated to give high education

u1 <- log(Il - Pars[5]) + (Pars[4] / (1 - Pars[4])) * log(Ih - Pars[5])

#Payoff of low educated to give low education

u2 <- (1 / (1 - Pars[4])) * log(Il)


#Average payoff function

w_bar <- w1 * State[1] + w2 * State[2]

u_bar <- u1 * State[3] + u2 * State[4]


#Differential Equations

dp1 <- State[1] * (w1 - w_bar)

dp2 <- State[2] * (w2 - w_bar)

dq1 <- State[3] * (u1 - u_bar)

dq2 <- State[4] * (u2 - u_bar)

dsH <- (- (State[2]) * State[5]) + ( State[3] * (1 - State[5]))
```

```
#Solutions

return(list(c(dp1, dp2, dq1, dq2,dsH)))

})

}
```