Homework Day 5 Solutions - ECON 186

Problem 1. Chiang and Wainwright 7.4 #2(a, c), 7

2)

a)

$$f_x = 2x + 5$$

$$f_y = 5x - 3y^2$$

c)

$$f_x = \frac{(x+y)(2) - (2x-3y)(1)}{(x+y)^2} = \frac{2x+2y-2x+3y}{(x+y)^2} = \frac{5y}{(x+y)^2}$$

$$f_y = \frac{(x+y)(-3) - (2x-3y)(1)}{(x+y)^2} = \frac{-3x-3y-2x+3y}{(x+y)^2} = -\frac{5x}{(x+y)^2}$$

7)

a)

$$grad\ f(x,y,z) = \left(2x, 3y^2, 4z^3\right)$$

b)

$$grad\ f(x,y,z) = (yz, xz, xy)$$

Problem 2. Chiang and Wainwright 7.6 #1

1)

a)

$$|J| = \begin{vmatrix} 6x_1 & 1 \\ (36x^2 + 12x_1x_2 + 48x_1) & (6x_1^2 + 2x_2 + 8) \end{vmatrix} = 0$$

So, the functions are dependent.

b)

$$|J| = \begin{bmatrix} 6x_1 & 4x_2 \\ 5 & 0 \end{bmatrix} = -20x_2$$

The determinant is nonzero, so the functions are independent.

Problem 3. Chiang and Wainwright 8.1 #6

6)

a) The price elasticity of demand is

$$\epsilon^d = \frac{\partial Q}{\partial P}\left(\frac{P}{Q}\right)$$

where the partial derivative with respect to price is

$$\frac{\partial Q}{\partial P} = -2$$

and $Q = 100 - 2(20) + 0.02(5000) = 160$. Therefore,

$$\epsilon^d = (-2)\,\frac{20}{160} = -\frac{1}{4}$$

b) The income elasticity of demand is

$$\eta = \frac{\partial Q}{\partial Y}\left(\frac{Y}{Q}\right)$$

where the partial derivative with respect to income is

$$\frac{\partial Q}{\partial Y} = 0.02$$

Substituting the relevant values

$$\eta = (0.02)\,\frac{5000}{160} = 0.625$$

Problem 4. Chiang and Wainwright 8.2 #7(a)

7)

a)

$$U_x = -15x^2 - 12y$$

$$U_y = -12x - 30y^4$$

$$dU = -\left(15x^2 + 12y\right) dx - \left(12x + 30y^4\right) dy$$

Problem 5. Chiang and Wainwright 8.3 #2

2)

a)

$$dy = \frac{(x_1 + x_2)\, dx_1 - x_1\,(dx_1 + dx_2)}{(x_1 + x_2)^2} = \frac{x_2 dx_1 - x_1 dx_2}{(x_1 + x_2)^2}$$

b)

$$dy = \frac{(x_1 + x_2)\,(2x_2 dx_1 + 2x_1 dx_2) - 2x_1 x_2\,(dx_1 + dx_2)}{(x_1 + x_2)^2} = \frac{2x_2^2 dx_1 + 2x_1^2 dx_2}{(x_1 + x_2)^2}$$

Problem 6. Chiang and Wainwright 8.4 #1

1)

a)

$$\frac{dz}{dy} = z_x \frac{dx}{dy} + z_y = (5 + y)\, 6y + x - 2y = 28y + 6y^2 + x = 28y + 9y^2$$

b)

$$\frac{dz}{dy} = 4y - \frac{8}{y^3}$$

c)

$$\frac{dz}{dy} = -15x + 3y = 108y - 30$$

Problem 7. Chiang and Wainwright 8.5 #2(a, c), 6

2)

a)

$$\frac{dy}{dx} = -\frac{f_x}{f_y} = -\frac{6x + 2y}{12y^2 + 2x}$$

c)

$$\frac{dy}{dx} = -\frac{f_x}{f_y} = -\frac{14x + 2y^2}{36y^3 + 4xy}$$

6) Point $(x = 1, y = 2, z = 0)$ satisfies the given equation. Since the three derivatives $F_x = 2x + 3y,\ F_y = 3x + 2z + 2y,\ F_z = 2y + 2z$ all exist and are continuous, and $F_z = 4 \neq 0$ at the given point, an implicit function $z = f(x, y)$ is defined. At the given point, we have

$$\frac{\partial z}{\partial x} = -\frac{2x + 3y}{2y + 2z} = -2$$

$$\frac{\partial z}{\partial y} = -\frac{3x + 2z + 2y}{2y + 2z} = -\frac{7}{4}$$