ECON 186 Class Notes: Introduction and Basics

Jijian Fan

Introduction

- In economics, we focus on answering economic questions by creating and testing models.
- Empirical
  - Data Analysis via the use of econometrics
- Theoretical
  - Step 1: Assumptions
  - Step 2: Set up the model
  - Step 3: Solve for equilibrium or optimization conditions
    - ★ Alternatively, numerical simulation if a closed form solution is not possible.
  - Step 4: Comparative Statics

Ingredients of a model

- Variables whose solution values we seek to obtain from the model are *endogenous variables* (originating from within).
- Variables whose values are determined outside the model and are taken as given in the model are *exogenous variables* (originating from outside the system).
- Constant: A magnitude that does not change.
  - Example: 5
- Parameter: A symbol that is representing a constant but is not actually assigned to any particular number.
  - Example: The symbol a could represent any integer in a given model.
- Note: The textbook uses subscript 0 to indicate exogenous variables.
  - Example: If P is price, then P_0 is a given exogenously determined price.

Introduction to Sets

- A set is a collection of distinct objects.
- I could assign each of you a number and create a set or take each of your heights and create a set.
- Examples of Enumeration

$$S = \{1, 2, 3, 4, 5\}$$

$$J = \{x | 2 < x < 5\}$$

- S is a finite set, while J is an infinite set.
- $2 \in S$ means that the integer 2 belongs to the set S while $2 \notin J$ means that the integer 2 does not belong to the set J.

Introduction to Sets

- Two sets are equal if they contain exactly the same elements. That is, two sets are equal if and only if they are subsets of each other.
- Suppose we have two sets J and K. Then J is a subset of K if every element in J is also in K.
  - Example: $J = \{x|2 < x < 5\}$ and $K = \{x|2 \leq x \leq 5\}$. Then $J \subset K$
  - Are J and K equal?

Introduction to Sets

- The largest possible subset of any set is itself. The smallest possible subset is the null set or empty set denoted by $\oslash$ or $\{\}$.
- Theorem: The null set is a subset of every set.
- Proof by contradiction: If $\oslash$ is not a subset of S, then $\oslash$ must contain at least one element x such that $x \notin S$. However, by definition, $\oslash$ has no elements, so we cannot say that $\oslash \not\subset S$, so $\oslash$ is a subset of S.
- Two sets with no common elements are disjoint.
  - Are J and K disjoint?

Operations on Sets

- A union of two sets A and B, represented by $A \cup B$ is a new set containing the elements in A or B.
- The intersection of two sets A and B, represented by $A \cap B$ is a new set containing the elements in both A and B.
- Example 1: Let $A = \{3, 5, 7\}$ and $B = \{2, 3, 4, 8\}$. Then $A \cup B = \{2, 3, 4, 5, 7, 8\}$ and $A \cap B = \{3\}$.
- Example 2: Let $A = \{-2, -1\}$ and $B = \{1, 2\}$. Then $A \cup B = \{-2, -1, 1, 2\}$ and $A \cap B = \{\oslash\}$.
- Let U be some set, which we call the universal set. Then, the complement of A, which we call $\widetilde{A}$ is the set that contains all the elements of U that are not in A.
- Example: Let $U = \{5, 6, 7, 8, 9\}$ and $A = \{5, 6\}$ then $\widetilde{A} = \{7, 8, 9\}$.
- Example: Let U be the set of all integers and A be the set of all even integers, then $\widetilde{A}$ is the set of all odd integers.

Laws of Set Operations

- Commutative law: $A \cup B = B \cup A$ and $A \cap B = B \cap A$.
- Associative Law: $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$
- Distributive Law: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cup C)$
- Let $A = \{4,5\}$, $B = \{3,6,7\}, C = \{2.3\}$. Verify the distributive law.
  - First part
    - Left: $A \cup (B \cap C) = \{4,5\} \cup \{3\} = \{3,4,5\}$
    - Right: $(A \cup B) \cap (A \cup C) = \{3,4,5,6,7\} \cap \{2,3,4,5\} = \{3,4,5\}$
  - Second part
    - Left: $A \cap (B \cup C) = \{4,5\} \cap \{2,3,6,7\} = \oslash$
    - Right: $(A \cap B) \cup (A \cup C) = \oslash \cup \oslash = \oslash$

Relations and Functions

- Ordered pairs, denoted by (a, b) and (b, a), have the property that $(a, b) \neq (b, a)$ unless $a = b$. Therefore, the order matters unlike the unordered pair $\{a, b\}$.
- The most prominent use is the Cartesian coordinate plane, which is made up of an infinite number of ordered pairs.
- Suppose $x = \{1, 2\}$ and $y = \{3, 4\}$ then consider all possible ordered pairs with the first element taken from x and the second element from y. These would be $(1, 3), (1, 4), (2, 3), (2, 4)$.
- This is called the Cartesian product which we write as $x \times y = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

Relations and Functions

- Any collection of ordered pairs, that is, subset of the Cartesian product, is a relation between y and x.
- For convenience, we will refer to the elements of $x \times y$ as (x, y).
- Example 1: The set $\{(x,y)|y = 2x\}$ consists of all ordered pairs lying on the line $y = 2x$.
- Example 2: The set $\{(x,y)|y \leq x\}$ corresponds to the set of all ordered pairs which satisfy the inequality $y \leq x$.
- A function, represented by $y = f(x)$ is a special case of a relation which has the property that any x value uniquely determines a y value.
  - x is the argument of the function, and y is the value of the function. In economics and particularly econometrics, we refer to as x as the independent variable and y as the dependent variable.
- Example 1 is a function, while Example 2 is not.

Types of Functions

- Constant functions are functions whose value stays the same regardless of the value of x.
- Example 1: $y = f(x) = 7$
- Example 2: $Y = C + I + G + NX = 300 + 200 + 150 - 100 = 550$
- Polynomial functions: $y = a_0 + a_1x + a_2x^2 + ... + a_nx^n$
  - $n = 1$: Linear function - 1st degree polynomial
  - $n = 2$: Quadratic function - 2nd degree polynomial
  - $n = 3$: Cubic function - 3rd degree polynomial
  - One of the primary uses for polynomial functions is fitting curves to scatter plots produced from data. Heavily used in econometrics.

Types of Functions

- Rational functions are functions where y is expressed as a ratio of two polynomials in x.
- Example: $y = \frac{a}{x}$ or $xy = a$. This function plots a rectangular-hyperbola. If we let $y = P$ and $x = Q$ then this function plots the demand curve where expenditure PQ is constant at all levels of price and the function is unitary elastic at all points.
- In addition to functions of one variable, we may also consider functions of two or more variables, such as $z = g(x, y)$. The domain in this case is a set of ordered pairs.
- Example 1: Suppose output is determined by capital (k) and labor (l), then the general production function is $Q = q(k, l)$.
- Example 2: Suppose utility is determined by consumption (c) and leisure (l), then the utility function can be written as $U = u(c, l)$.

Equilibrium and Systems of Equations

- An economic equilibrium is a state where economic forces are balanced. That is, all economic variables in the model will be unchanged from their equilibrium values in the absence of external influences.
- Example: The equilibrium for a single product is where the demand and supply curve for that product intersect.
  - Suppose the demand curve for a good is $Q_d = 5 - 2P$, the supply curve is $Q_s = 2 + P$, and the market clearing condition or equilibrium condition is $Q_d = Q_s$. What is the equilibrium price?

$$Q_d = Q_s \rightarrow 5 - 2P = 2 + P \rightarrow 3 = 3P \rightarrow P = 1$$

  - This is an example of partial equilibrium, that is, it takes into account only part of the market (or a market with just one good), in this case, the market for just one good. This equilibrium does not take into account how changes in prices of other goods may affect this particular good.

General Equilibrium

- General equilibrium theory attempts to determine the equilibrium values of variables in an entire economy with more than one interacting market.
- Example: Suppose we have an economy with only two goods described by the following equations:

$$1) Q_{d1} = 10 - 2P_1 + P_2$$

$$2) Q_{s1} = -2 + 3P_1$$

$$3) Q_{d2} = 15 + P_1 - P_2$$

$$4) Q_{s2} = -1 + 2P_2$$

$$5) Q_{d1} = Q_{s1}$$

$$6) Q_{d2} = Q_{s2}$$

Solving the Example

- Use 5) to set supply and demand equal for the first good.

$$10 - 2P_1 + P_2 = -2 + 3P_1 \rightarrow 5P_1 - 12 = P_2$$

- Use 6) to set supply and demand equal for the second good.

$$15 + P_1 - P_2 = -1 + 2P_2 \rightarrow 16 + P_1 = 3P_2 \rightarrow P_2 = \frac{16}{3} + \frac{P_1}{3}$$

- Set both equations equal

$$5P_1 - 12 = \frac{16}{3} + \frac{P_1}{3} \rightarrow \frac{15P_1}{3} - \frac{36}{3} = \frac{16}{3} + \frac{P_1}{3} \rightarrow \frac{14P_1}{3} = \frac{52}{3} \rightarrow$$

$$P_1^* = \frac{52}{14} = \frac{26}{7}$$

  - Substitute back in:

$$P_2^* = 5(\frac{26}{7}) - 12 = \frac{130}{7} - \frac{84}{7} = \frac{46}{7}$$