Homework Day 2 Solutions - ECON 186

Problem 1. Chiang and Wainwright 4.2 #1,2, 4, 6

1)

a) $\begin{bmatrix} 7 & 3 \\ 9 & 7 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 4 \\ 0 & -8 \end{bmatrix}$

c) $\begin{bmatrix} 21 & -3 \\ 18 & 27 \end{bmatrix}$

d) $\begin{bmatrix} 16 & 22 \\ 24 & -6 \end{bmatrix}$

2)

a) Yes it is defined.

$$AB = \begin{bmatrix} 28 & 64 \\ 6 & 0 \\ 13 & 8 \end{bmatrix}$$

No, BA does not exist because the product is not conformable.

b) Both are defined, but they are not equal.

$$BC = \begin{bmatrix} 14 & 4 \\ 69 & 30 \end{bmatrix}$$

$$CB = \begin{bmatrix} 20 & 16 \\ 21 & 24 \end{bmatrix}$$

4)

a) $\begin{bmatrix} 0 & 2 \\ 36 & 20 \\ 16 & 3 \end{bmatrix}$

(3×2)

b) $\begin{bmatrix} 49 & 3 \\ 4 & 3 \end{bmatrix}$
(2×2)

c) $\begin{bmatrix} 3x + 5y \\ 4x + 2y - 7z \end{bmatrix}$
(2×1)

d) $\begin{bmatrix} 7a + c & 2b + 4c \end{bmatrix}$
(1×2)

6)

a) $x_2 + x_3 + x_4 + x_5$

b) $a_5x_5 + a_6x_6 + a_7x_7 + a_8x_8$

c) $b\left(x_1 + x_2 + x_3 + x_4\right)$

d) $a_1x^0 + a_2x^1 + ... + a_nx^{n-1} = a_1 + a_2x + a_3x^2 + ... + a_nx^{n-1}$

e) $x^2 + (x+1)^2 + (x+2)^2 + (x+3)^2$

Problem 2. Chiang and Wainwright 4.3 #1(a, b, d), 2

1)

a)
$$uv' = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 15 & 5 & -5 \\ 3 & 1 & -1 \\ 9 & 3 & -3 \end{bmatrix}$$

b)
$$uw' = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 7 & 5 & 8 \end{bmatrix} = \begin{bmatrix} 35 & 25 & 40 \\ 7 & 5 & 8 \\ 21 & 15 & 24 \end{bmatrix}$$

d)
$$\begin{bmatrix} 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 15 + 1 - 3 \end{bmatrix} = [13] = 13$$

2)

a) All are defined except $w'x$ and $x'y'$.

b)

$$xy^{'} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} y_1 & y_2 \end{bmatrix} = \begin{bmatrix} x_1y_1 & x_1y_2 \\ x_2y_1 & x_2y_2 \end{bmatrix}$$

$$xy^{'} = \begin{bmatrix} y_1 & y_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = y_1^2 + y_2^2$$

$$zz^{'} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \begin{bmatrix} z_1 & z_2 \end{bmatrix} = \begin{bmatrix} z_1^2 & z_1z_2 \\ z_2z_1 & z_2^2 \end{bmatrix}$$

$$yw^{'} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \begin{bmatrix} 3 & 2 & 16 \end{bmatrix} = \begin{bmatrix} 3y_1 & 2y_1 & 16y_1 \\ 3y_2 & 2y_2 & 16y_2 \end{bmatrix}$$

$$x \bullet y = x_1y_1 + x_2y_2$$

Problem 3. Chiang and Wainwright 4.4 #1

1)

a)

$$(A + B) + C = A + (B + C) = \begin{bmatrix} 5 & 17 \\ 11 & 17 \end{bmatrix}$$

b)

$$(A + B) + C = A + (B + C) = \begin{bmatrix} -1 & 9 \\ 9 & -1 \end{bmatrix}$$

Problem 4. Chiang and Wainwright 4.5 #1

1)

a)

$$AI_3 = \begin{bmatrix} -1 & 5 & 7 \\ 0 & -2 & 4 \end{bmatrix}$$

b)

$$I_2A = \begin{bmatrix} -1 & 5 & 7 \\ 0 & -2 & 4 \end{bmatrix}$$

c)

$$I_2 x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

d)

$$x^{'} I_2 = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$$

Problem 5. Chiang and Wainwright 4.6 #1, 2

1)

$$A^{'} = \begin{bmatrix} 0 & -1 \\ 4 & 3 \end{bmatrix}$$

$$B^{'} = \begin{bmatrix} 3 & 0 \\ -8 & 1 \end{bmatrix}$$

$$C^{'} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \\ 9 & 1 \end{bmatrix}$$

2)

a)

$$(A + B)^{'} = A^{'} + B^{'} = \begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix}$$

b)

$$(AC)^{'} = C^{'} A^{'} = \begin{bmatrix} 24 & 17 \\ 4 & 3 \\ 4 & -6 \end{bmatrix}$$